1st Week Friday

24 Şubat 2023 Cuma 10:33

Strict Triangular Form

Ex

$x_1 = ---$

$$\begin{aligned} 3x_1 + 2x_2 - x_3 &= -2 \\ x_2 - 2x_3 &= 3 \\ 2x_3 &= 4 \end{aligned}$$

$x_3 = ---$

⇒ $x_3 = 2$ substitude

$x_2 - 2.2 = 3 \Rightarrow x_2 = 7$

$3x_1 + 2.7 - 2 = -2 \Rightarrow x_1 = -14/3$

⇒ $(-14/3, 7, 2)$ → the unique solution for the system.

1st linear eqn.

→ $a_{22}x_2$ 2nd   strict triangular form

2D arrays of real numbers

System of Linear Eqns ⇄ Matrices

$$\begin{array}{c} \text{1st column} \quad \text{2nd column} \\ \begin{bmatrix} - & - & - & - & \cdots \\ - & - & - & - & \cdots \\ \vdots \\ - & - \end{bmatrix} \begin{array}{l} \rightarrow \text{1st row} \\ \rightarrow \text{2nd row} \\ \vdots \end{array} \end{array}$$

m rows, n columns → m×n matrix

a_{ij} → the coeff of the j^{th} variable (x_j) in the i^{th} eqn

Ex/ $\begin{bmatrix} 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 \end{bmatrix}$ 2×4 → # of columns; ↓ # of rows

1st eqn → $a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$

2nd eqn → $a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$

⋮

mth eqn → $a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m$

m eqns

n variables $x_1, x_2, \dots, x_n$

m×n system of linear eqns

Augmented Matrix of a System of LE

$$\begin{array}{r|ccccc|c|} & \text{1st column} & \text{2nd column} & & & & \\ \text{1st row} \rightarrow & a_{11} & a_{12} & a_{13} & \cdots & a_{1n} & b_1 \\ \text{2nd row} \rightarrow & a_{21} & a_{22} & a_{23} & \cdots & a_{2n} & b_2 \\ \vdots & & & & & & \vdots \\ \text{mth row} \rightarrow & a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} & b_m \end{array}$$

coefficents

results column

→ m rows, n+1 column

$m \times (n+1)$ matrix

($A = [a_{11} \quad -- \quad a_{1n}]$)

Coefficents

column

$$\left(A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ & & \end{bmatrix}_{m\times n} \right)$$

coefficient matrix

Ex/

$$\begin{aligned} 3x_1 + 2x_2 - 1x_3 &= -2 \\ 0x_1 + 1x_2 - 2x_3 &= 3 \\ 0x_1 + 0x_2 + 2x_3 &= 4 \end{aligned} \longrightarrow$$

The augmented matrix of this system is

$$\left[\begin{array}{ccc|c} 3 & 2 & -1 & -2 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 2 & 4 \end{array}\right]$$

in → strict triangular form

Ex/

$$\begin{aligned} 3x_1 + 2x_2 - x_3 &= -2 \quad \rightarrow \text{1st linear eqn.} \\ -3x_1 - x_2 - x_3 &= 5 \quad \rightarrow \text{2nd lin.} \\ 3x_1 + 2x_2 + x_3 &= 2 \end{aligned}$$

↓ the augmented matrix of the system

Allowed Operations to create "Equivalent" Systems !

! 1) changing the order of any two eqns

! 2) multiplying one eqn with a real number

3) $c r_i + r_j \longrightarrow r_j$! $c \in \mathbb{R}$

Ex/

$$\begin{array}{c} r_1 \rightarrow \\ r_2 \rightarrow \\ r_3 \rightarrow \end{array} \left[\begin{array}{ccc|c} 3 & 2 & -1 & -2 \\ -3 & -1 & -1 & 5 \\ 3 & 2 & 1 & 2 \end{array}\right] \xrightarrow{r_2 \leftrightarrow r_3} \left[\begin{array}{ccc|c} 3 & 2 & -1 & -2 \\ 3 & 2 & 1 & 2 \\ -3 & -1 & -1 & 5 \end{array}\right]$$

→ an equivalet system to the original one.

Ex/

$$r_1 \rightarrow \left[\begin{array}{ccc|c} 3 & 2 & -1 & -2 \\ -3 & -1 & -1 & 5 \\ 3 & 2 & 1 & 2 \end{array}\right] \xrightarrow{\frac{1}{3} \cdot r_1 \rightarrow r_1} \left[\begin{array}{ccc|c} 1 & 2/3 & -1/3 & -2/3 \\ -3 & -1 & -1 & 5 \\ 3 & 2 & 1 & 2 \end{array}\right]$$

→ an equivant system to the original one.

↓ $2r_3 \rightarrow r_3$

$$\left[\begin{array}{ccc|c} 1 & 2/3 & -1/3 & -2/3 \\ -3 & -1 & -1 & 5 \\ 6 & 4 & 2 & 4 \end{array}\right]$$

→ an equivalent system to the original one.

↓ $r_1 \leftrightarrow r_2$

$$\left[\begin{array}{ccc|c} -3 & -1 & -1 & 5 \\ 1 & 2/3 & -1/3 & -2/3 \\ 6 & 4 & 2 & 4 \end{array}\right]$$

→ //

Ex

$$\left[\begin{array}{ccc|c} 3 & 2 & -1 & -2 \\ -3 & -1 & -1 & 5 \\ 3 & 2 & 1 & 2 \end{array}\right] \xrightarrow{2r_1+r_2\to \overset{!}{r_2}} \left[\begin{array}{ccc|c} 3 & 2 & -1 & -2 \\ 3 & 3 & -3 & 1 \\ 3 & 2 & 1 & 2 \end{array}\right]$$

→ an equiv. syste

$$\left.\begin{array}{lcccc} 2r_1 & 6 & 4 & -2 & -4 \\ r_2 & -3 & -1 & -1 & 5 \end{array}\right\} \to \begin{array}{cccc} 3 & 3 & -3 & 1 \end{array} \to r_2$$

Ex

$$\left[\begin{array}{ccc|c} 3 & 2 & -1 & -2 \\ -3 & -1 & -1 & 5 \\ 3 & 2 & 1 & 2 \end{array}\right] \xrightarrow{1.r_1+r_2\to r_2} \left[\begin{array}{ccc|c} 3 & 2 & -1 & -2 \\ 0 & 1 & -2 & 3 \\ 3 & 2 & 1 & 2 \end{array}\right]$$

to make this zero
use multiples of the upper row.

to make this zero
use the nonzero upper rows.

$$\xrightarrow{-1r_1+r_3\to r_3} \left[\begin{array}{ccc|c} 3 & 2 & -1 & -2 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 2 & 4 \end{array}\right]$$

→ an equiv.

Ex

3) $c.r_i + r_j \to r_j$ ← different r_j's affected ✓
same r_i is used ✓

$$c\left[\begin{array}{ccc|c} 3 & 2 & -1 & -2 \\ -3 & -1 & -1 & 5 \\ 3 & 2 & 1 & 2 \end{array}\right] \xrightarrow[\substack{1r_1+r_2\to r_2 \\ -1r_1+r_3\to r_3}]{} \left[\begin{array}{ccc|c} 3 & 2 & -1 & -2 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 2 & 4 \end{array}\right]$$